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SUMMARY OF RESEARCH

(May 1, 1966 - October 31, 1966)

The major goals of this research are to generate a deep understanding of communication channels and sources and to use this understanding in the development of reliable, efficient communication techniques.

1. Convolutional Codes and Decoding

A technique has been investigated for combining block coding with convolutional coding.[1] By using a combination of sequential decoding and algebraic decoding, it has been demonstrated that reliable communication can be achieved at all rates below channel capacity. While this technique is more complicated than sequential decoding alone, it operates at higher rates and has less buffer storage requirements than sequential decoding.

Preliminary results have been achieved on the relative error probability performance of nonsystematic versus systematic convolutional codes.[2] Surprisingly, the results indicate that for nonsystematic codes, the exponential decay of error probability with code constraint length is much faster than for systematic codes.

A theoretical investigation has been undertaken to assess the capabilities of tree codes for error-correction purposes, fixed convolutional codes being treated as an important special case. Minimum distances of an appropriate type have been defined for both feedback decoding and nonfeedback, or definite decoding, of tree codes. Some new upper and lower bounds on minimum distance have been obtained, and effort in this direction continues. The error-propagation effect, resulting from feedback decoding, is also being investigated. A new decoding technique, called semidefinite decoding, which has some of the features of feedback decoding but avoids the error-propagation effect, is being simulated for a wide variety of convolutional codes on the binary symmetric channel to determine whether it offers an over-all advantage over feedback decoding.

2. Optical Communication

The extension of communication theory to optical channels has focused upon the effects of atmospheric turbulence. A model that is appropriate for the analysis of communication systems has been developed with the techniques of geometric optics.[3] Since the approach that is being taken provides some insights into more general problems of atmospheric propagation, it is being developed beyond the level required for communication theory to its natural conclusion.

A 4-km one-way propagation path operating at 6328 Å has been established with the cooperation of the Harvard College Observatory.[4] The facility has been used to investigate the statistical properties of intensity, and is now being modified for round-trip transmission. An interferometric system has also been developed to study turbulence-induced phase fluctuations.[5] Theoretical and experimental studies concerning depolarization caused by turbulence has led to the conclusion that depolarization is negligible.[6] The experimental aspects of the study were performed at Bell Telephone Laboratories, Inc., Crawford Hill.

Recently initiated investigations include studies of the communication reliability of free space and atmospheric channels in the absence of quantum effects, the reliability of quantum channels, the potential of forward-scatter communication systems, and the estimation of incoherently illuminated objects viewed through a turbulent atmosphere.

3. Specific Channels and Coding

One of the major types of channels that are being investigated is a class of fading dispersive channels such as HF and Tropo. Particular emphasis is being placed upon the situation in which the information rate is comparable to, or exceeds, the available bandwidth; the complementary situation has been considered previously.[7]

The performance that can be achieved with one particular signaling scheme when both the coherence bandwidth and the frequency dispersion of the channel are small has been analytically and experimentally investigated.[8] The results suggest that a very large energy-to-noise ratio is required for satisfactory communication when the rate is comparable with the bandwidth. A comprehensive analytical study of the attainable reliability is now in progress.[9]

A number of coding theorems have been developed for statistically related parallel channels in the absence of cross talk.[10] Such models can be applied, for example, to frequency-multiplexed channels, and also yield insight into the behavior of channels with memory.

A theoretical investigation of the use of coding on unsynchronized, noisy channels is in progress.[11] The lack of synchronization does not change the random-coding exponent, but appears to radically change the exponent of the error probability at low rates or for channels with little noise.

4. Source Coding with a Distortion Measure

The interrelations between source and channel coding have been investigated for discrete memoryless sources and channels.[12] The results indicate that for combined source and channel coding with block length n, the theoretical minimum distortion is approached with increasing n, the dependence of the rate of approach with n being between $1/n$ and $\sqrt{\ln n/n}$.

In another investigation, a distortion measure for a discrete source was considered with a distortion of 1 for error, and 0 for no error. This led to a complete solution for the minimum probability of error achievable for a discrete memoryless source when transmitting over a channel with capacity less than the source entropy.[13]

Further investigations are being made on techniques for source coding and on the effect of quantizers in source coding.

5. Coding and the Processing of Information

The problem of performing reliable computation with unreliable computing elements has been reviewed by Winograd and Cowan.[14] Earlier work in this field has led to results that permit the reliability of computation to be improved only by increasing the number of unreliable elements used per unit computation or increasing the complexity of each such element without reducing its reliability. Results analogous to the noisy channel coding theorem of information theory, which would permit increasing the reliability of computation while performing more of it at once, with a fixed redundancy of equipment per unit computation, have not been available. Michael C. Taylor[15] has shown that application of low-density parity-check codes[16] to the problem of reliable storage of information in a noisy register leads to a result of coding-theorem character: Doubling both the number of noisy components and the amount of information stored reduces the probability of error, or, alternatively, increases the mean time until an error occurs.

References

1. D. Falconer, "A Hybrid Sequential and Algebraic Decoding Scheme," Ph.D. Thesis, Department of Electrical Engineering, M.I.T., September 1966.
2. E. Bucher, "Error Probability for Systematic Convolution Codes," S.M. Thesis, Department of Electrical Engineering, M.I.T., Feb. 1967.
3. R. S. Kennedy and E. V. Hoversten, "Optical Propagation through a Turbulent Atmosphere," Quarterly Progress Report No. 84, Research Laboratory of Electronics, M.I.T., January 15, 1967, pp. 212-224.
4. J. E. Roberson, "A Study of Atmospheric Effects on Intensity Spatial and Temporal Properties at 6328 Å," S.M. Thesis, Department of Electrical Engineering, M.I.T., October 1966.
5. R. Yusick, "Interferometric Measurement of Optical Phase Noise over Atmospheric Paths," S.M. Thesis, Department of Electrical Engineering, M.I.T., October 1966.
6. A. A. M. Saleh, "Laser Wave Polarization by Atmospheric Transmission," S.M. Thesis, Department of Electrical Engineering, M.I.T., November 1966.
7. R. S. Kennedy, "Fading Dispersive Communication Channels" (to be published by John Wiley and Sons, Inc.)
8. J. Moldon, "High Rate Reliability over Fading Dispersive Communication Channels," S.M. Thesis, Department of Electrical Engineering, M.I.T., June 1966.
9. J. Richters, "Low-Rate Upper Bounds on Error Probability for Fading Dispersive Channels," Quarterly Progress Report No. 84, Research Laboratory of Electronics, M.I.T., January 15, 1967, pp. 207-211.
10. J. Max, "Parallel Channels without Cross Talk," Sc.D. thesis research.
11. D. Chase, "Communication over Noisy Channels with no a priori Synchronization Information," Sc.D. thesis research.
12. R. Pilc, "Coding Theorems for Discrete Source-Channel Pairs," Ph.D. Thesis, Department of Electrical Engineering, M.I.T., November 1966.

13. J. T. Pinkston, "An Application of Rate Distortion Theory to a Converse to the Coding Theorem," submitted for publication to IEEE Trans. on Information Theory.

14. S. Winograd and J. C. Cowan, Reliable Computation in the Presence of Noise (The M.I.T. Press, Cambridge, Mass., 1963).

15. M. C. Taylor, "Randomly Perturbed Computer Systems," Ph.D. Thesis, Department of Electrical Engineering, M.I.T., September 1966.

16. R. G. Gallager, Low Density Parity Check Codes (The M.I.T. Press, Cambridge, Mass., 1963).